Acceleration-Dependent Fluid Forces

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I. Introduction

THE complete equations of motion of bodies in a fluid contain acceleration-dependent fluid forces that are required for a wide variety of engineering problems. These forces are significant for bodies moving in water, whether fully or partially immersed (ships, submarines, underwater weapon systems), lighter-than-air systems (airships, tethered aerostats, scientific balloons), and parachutes. These forces are also important in the stability and control analyses of missiles[2] and other solid-fluid dynamic interaction problems (such as fuel sloshing, fluid forces on dams, wind forces on buildings and bridges, blood flow, head and eye injury problems). The literature is vast on the subject, and one really does not know where to begin citing it. Most of the available references deal with a specific problem or a technique emphasizing a few aspects.[3-5]

The objective of this Note is to present a comprehensive formulation of the treatment of acceleration-dependent forces of rigid as well as flexible bodies that is efficient for digital computer calculations. The objective is motivated by the following facts:

1) There are several erroneous concepts and applications in the literature.[6]

2) Finite-element methods[7,8] are not numerically efficient for the problems involving infinite domains.

3) There is a need for a comprehensive formulation that can be used to develop a stand-alone state-of-the-art digital computer program to treat rigid as well as flexible bodies.

II. Acceleration-Dependent Fluid Forces

Small- as well as large-disturbance, steady and unsteady, incompressible potential flows are governed by Laplace's equation[9]

$$\nabla^2\phi=0 \tag{1}$$

where ϕ is the velocity potential. The boundary conditions for the potential flow problem can be written[9]

$$\frac{\partial F}{\partial t}+Q\cdot\text{grad } F=0 \tag{2}$$

where $F(x,y,z,t)=0$ is the equation for the surface of the body and $Q(=\text{grad}\phi)$ is the total velocity vector of the fluid. The fluid forces experienced by a body as given by an incompressible potential analysis are given in Ref. 1 and, for the sake of completeness, an independent and brief derivation is presented below.

Consider a region R, which is enclosed by a surface S, which contains fluid in motion. The kinetic energy T of the fluid in R is given by

$$T=\iiint_R \frac{\rho Q^2}{2}\mathrm{d}R=\iiint_R \frac{\rho(\text{grad}\phi\cdot\text{grad}\phi)}{2}\mathrm{d}R \tag{3}$$

where ρ is the density of the fluid. From the first form of Green's theorem and Eq. (1), the following identity can be written for harmonic functions.

$$\iiint_R (\text{grad}\phi\cdot\text{grad}\phi)\mathrm{d}R=\iint_S \phi\frac{\partial\phi}{\partial n}\mathrm{d}S \tag{4}$$

where n indicates the normal direction. Substituting Eq. (4) into Eq. (3)

$$T=\frac{1}{2}\iint_S \rho\phi\frac{\partial\phi}{\partial n}\mathrm{d}S \tag{5}$$

one can assume the following functional form for ϕ

$$\phi=\sum_{i=1}^{6} u_i(t)\phi_i(x,y,z) \tag{6}$$

where u_1, u_2, and u_3 are linear velocities and u_4, u_5, and u_6 are angular velocities about x,y,z axes, respectively. The above form is a feasible solution because

1) The governing equation is linear.

2) The boundary condition is linear for problems under consideration.

3) Problem admits solutions that are separable in time and space.

The substitution of Eq. (6) into Eq. (5) and the rearrangement of the resulting equation yields

$$T=\tfrac{1}{2}\{u_i\}^T[M_{ij}]\{u_i\} \tag{7}$$

where

$$M_{ij}=M_{ji}=\rho\iint_S \phi_i\frac{\partial\phi_j}{\partial n}\mathrm{d}S \tag{8}$$

The equations of motion of a rigid body referred to an arbitrary system of axes are given by[10]

$$\{F_i\}=\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{\partial T}{\partial u_i}\right\}+[W]\left\{\frac{\partial T}{\partial u_i}\right\} \tag{9}$$

$$\{M_i\}=\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{\partial T}{\partial u_j}\right\}+[W]\left\{\frac{\partial T}{\partial u_j}\right\}+[V]\left\{\frac{\partial T}{\partial u_i}\right\} \tag{10}$$

where $\{F_i\}$ = applied forces along the xyz axes, $\{M_i\}$ = applied moments about the xyz axes, $i=1,2,3$, and $j=4,5,6$.

$$[V]=\begin{bmatrix}0 & -w & v\\ w & 0 & -u\\ -v & u & 0\end{bmatrix},\quad [W]=\begin{bmatrix}0 & -r & q\\ r & 0 & -p\\ -q & p & 0\end{bmatrix}$$

$$u=u_1,\ v=u_2,\ w=u_3,\ u_4=p,\ u_5=q,\ u_6=r$$

Substituting Eq. (7) into Eqs. (9) and (10) yields the following equation for the fluid force, experienced by a body as given by an incompressible potential flow anlaysis.

$$\{F\}=[M]\{\dot{Q}\}+[T][M]\{Q\} \tag{11}$$

where

$\{F\}^T=\lfloor F_1\ F_2\ F_3\ M_1\ M_2\ M_3\rfloor$

$[M]$ = added mass matrix whose elements are defined by Eq. (8)

$\{Q\}^T=\lfloor u\ v\ w\ p\ q\ r\rfloor$

$[T]=\left[\begin{array}{c|c} W & 0\\ \hline V & W\end{array}\right]$, the submatrices $[V]$ and $[W]$ are defined in Eq. (10) and (0) is a null matrix

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III. Formulation for Rigid Bodies

The unsteady Bernoulli's equation for incompressible potential flows can be written as

$$\frac{P}{\rho}+\frac{Q^2}{2}+\frac{\partial\phi}{\partial t}=\text{const} \tag{12}$$

To determine the acceleration-dependent forces, the velocity term in the above equation is dropped and the constant term is set to zero. Hence Eq. (12) becomes

$$\frac{P}{\rho}+\frac{\partial\phi}{\partial t}=0 \tag{13}$$

By differentiating Eq. (1) with respect to time and substituting Eq. (10), the following equation is obtained for pressure

$$\nabla^2 P=0 \tag{14}$$

The boundary condition given by Eq. (2) can be simplified to the following equation

$$\frac{\partial P}{\partial n}=-\rho a_{ns} \tag{15}$$

where a_{ns} is the acceleration of the body normal to the surface.

If the linear accelerations $(\dot u,\dot v,\dot w)$ and angular accelerations $(\dot p,\dot q,\dot r)$ are specified as $\dot u=1$; $\dot v=\dot w=\dot p=\dot q=\dot r=0$, then the pressure distribution can be obtained by solving Eqs. (14) and (15). In solving this problem the unit acceleration $\dot u$ has to be resolved in the normal direction according to Eq. (15). By integrating the pressure associated with these boundary conditions ($\dot u=1$; $\dot v=\dot w=\dot p=\dot q=\dot r=0$), the forces and moments defined in Eq. (11) can be obtained. These forces are related to added mass coefficients as shown below:

$$F_1=M_{11},\ F_2=M_{12},\ F_3=M_{13}$$

$$M_1=M_{14},\ M_2=M_{15},\ M_3=M_{16}$$

Similarly, by specifying different sets of body accelerations, the remaining added mass coefficients of an arbitrary three-dimensional body can be determined. The sets of problems to be solved to determine the 21 added mass coefficients are given below.

Accelerations	Added Mass Coefficients
$\dot v=\dot w=\dot p=\dot q=\dot r=0,\ \dot u=1$	$M_{11},\ M_{12},\ M_{13},\ M_{14},\ M_{15},\ M_{16}$
$\dot u=\dot w=\dot p=\dot q=\dot r=0,\ \dot v=1$	$M_{22},\ M_{23},\ M_{24},\ M_{25},\ M_{26}$
$\dot u=\dot v=\dot p=\dot q=\dot r=0,\ \dot w=1$	$M_{33},\ M_{34},\ M_{35},\ M_{36}$
$\dot u=\dot v=\dot w=\dot q=\dot r=0,\ \dot p=1$	$M_{44},\ M_{45},\ M_{46}$
$\dot u=\dot v=\dot w=\dot p=\dot r=0,\ \dot q=1$	$M_{55},\ M_{66}$
$\dot u=\dot v=\dot w=\dot p=\dot q=0,\ \dot r=1$	M_{66}

IV. Formulation for Flexible Bodies

The main difference between a flexible body and a rigid body is that the flexible body will have infinite degrees of freedom, whereas the rigid body has six. The approach taken here is to divide the flexible body into many rigid bodies. A formulation is presented here to calculate the linear added mass terms of a three-dimensional flexible body and can be extended easily to calculate the nonlinear added mass terms. A source strength σ per unit area is assumed to be distributed over the surface of the body and the velocity potential at P_1 due to a source at Q_1 is given by

$$\phi(P_1)=-\frac{1}{4\pi}\iint\frac{\sigma(Q_1)}{R}\mathrm{d}A \tag{16}$$

where $R=|r_{P_1}-r_{Q_1}|$ and r_{P_1}, r_{Q_1} are position vectors of P_1 and Q_1. The position vectors r_{p_1} and r_{Q_1} are not functions of time but basically depend on the geometry of the body. The surface of the body is assumed to be divided into N panels, and Eq. (16) is written as summation of integrals for N panels as shown below

$$\phi(P_1)=-\frac{1}{4\pi}\sum_{j=1}^{N}\iint_{A_j}\frac{\sigma_j(Q_1)\mathrm{d}A_j}{R} \tag{17}$$

The weighted normal velocity of the ith panel is given by

$$V_{ni}=\frac{1}{A_i}\iint_{A_i}(\mathrm{grad}\phi\cdot\boldsymbol{n}_i)\mathrm{d}A_i \tag{18}$$

where n_i is the unit outward-drawn normal vector of the ith panel and A_i is the area of the ith panel. Substituting Eq. (17) into Eq. (18)

$$V_{ni}=\frac{1}{4\pi A_i}\iint_{A_i}\sum_{j=1}^{N}\iint_{A_j}\frac{\sigma_j(Q)(\boldsymbol{u}_{pq}\cdot\boldsymbol{n}_i)}{R^2}\mathrm{d}A_j\mathrm{d}A_i \tag{19}$$

where

$$\boldsymbol{u}_{pq}=(\boldsymbol{r}_{P_1}-\boldsymbol{r}_{Q_1})/R$$

Equation (19) can be written into a matrix equation of the following form

$$\{V_n\}=[A]\{\sigma\} \tag{20}$$

where

$$A_{ij}=\frac{1}{4\pi A_i}\iint_{A_i}\iint_{A_j}\left(\frac{\boldsymbol{u}_{pq}\cdot\boldsymbol{n}_i}{R^2}\right)\mathrm{d}A_j\mathrm{d}A_i$$

The forces and moments acting on the ith panel can be obtained from

$$\boldsymbol{F}=-\iint_{A_i}P\boldsymbol{n}_i\mathrm{d}A_i \tag{21}$$

$$\boldsymbol{M}=-\iint_{A_i}(\boldsymbol{r}\times P\boldsymbol{n}_i)\mathrm{d}A_i \tag{22}$$

By substituting Eqs. (13) and (17) into Eqs. (21) and (22),

$$\boldsymbol{F}=-\frac{\rho}{4\pi}\iint_{A_i}\sum_{j=1}^{N}\iint_{A_j}\frac{\dot\sigma_j\boldsymbol{n}_i\mathrm{d}A_j}{R}\mathrm{d}A_i \tag{23}$$

$$\boldsymbol{M}=-\frac{\rho}{4\pi}\iint_{A_i}\sum_{j=1}^{N}\iint_{A_j}\frac{\dot\sigma_j(\boldsymbol{r}\times\boldsymbol{n}_i)\mathrm{d}A_j}{R}\mathrm{d}A_i \tag{24}$$

Equations (23) and (24) can be written into matrix form as

$$\left\{\begin{array}{c}F\\ \hdashline M\end{array}\right\}=\left[\begin{array}{c}B\\ \hline C\end{array}\right]\{\dot\sigma\} \tag{25}$$

where the left-hand side vector contains the components of forces and moments in the panel coordinate system. Substituting Eq. (20) into Eq. (25) yields

$$\underset{6N\times 1}{\begin{Bmatrix} F \\ \hline M \end{Bmatrix}} = \underset{6N\times N}{\begin{bmatrix} B \\ \hline C \end{bmatrix}} \underset{N\times N}{[A]^{-1}} \underset{N\times 1}{\{\dot{V}_n\}} \tag{26}$$

For solid fluid interaction problems, it is useful to relate the panel accelerations to node acceleration. For instance, assume that the normal acceleration of the ith panel is related to the translational ($\dot{V}_{ij}$) and angular acceleration ($\dot{R}_{ij}$) of the jth node of the same panel by the following relation

$$\{\dot{V}_n\}_i = [T]\begin{Bmatrix} \dot{V} \\ \dot{R} \end{Bmatrix}_{ij} \tag{27}$$

Then, the associated added mass forces are given by substituting Eq. (27) into Eq. (26). The coefficient matrix of $\{\dot{V}_n\}$ in the resulting equation is the added mass matrix of the system.

V. Conclusions

A comprehensive approach for the treatment of acceleration-dependent fluid forces is presented. The methoid is very convenient for programming for digital computer calculations. The other significant advantages of the appraoch are as follows:

1) The approach gives the added mass distribution in addition to the gross coefficients.

2) The approach is applicable for arbitrary three-dimensional bodies; rigid as well as elastic.

3) The linear and nonlinear terms can be calculated.

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Optical Constants of Propellant-Grade Ammonium Perchlorate

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Introduction

CONTROLLING radiative heating of aluminum in composite propellants has recently been brought to light as a possible mechanism for reducing unwanted agglomeration of aluminum during the combustion of aluminized solid propellants.[1] In order to model the radiative scattering and absorption processes within the composite solid propellant, it is necessary to know the radiative properties of the components of the propellant. This Note reports the optical constants n and k of ammonium perchlorate (AP), one of the most common oxidizer materials used in composite solid propellants.

Previous spectroscopic studies[2,3] have been successful at revealing information about the structure and bonding of pure AP. However, these studies do not indicate the absolute magnitude of the optical constants of AP, but rather only the relative absorptivity. Furthermore, these studies deal with pure AP. No studies in the literature report the optical properties of AP used in composite solid propellants. This AP differs from pure AP in that it has, typically, been coated with the anticaking agent tricalcium phosphate (TCP). This small amount of impurity may have some influence on the optical properties of the AP.

Procedure

There are many ways to determine the optical constants of solids.[4,5] In this study, measurements of the near-normal reflectance were made from thin, plane AP disks, over the range of 2.6 to 9.6 μm. The disks were prepared from AP powder. A dispersion equation curve-fitting technique was then applied to find values of n and k consistent with the near-normal reflectance measurements.

To prepare the disks, 100 mg of AP powder was heated in an oven at a temperature of 120–130°C for 8 h to rid the powder of its moisture content. The dry powder was then placed inside a 1.27 cm diam die and a force of 84.5 kN (19,000 lbf) was applied for 5 min to the AP powder. The stainless steel pressing surfaces of the die were optically smooth to form smooth, specular sample surfaces. In addition, the die was evacuated at 25 mm of Hg during the pressing process to prevent air from being trapped inside the disks. The AP disks thus obtained were slightly milky. They appeared specularly reflecting, although some inhomogeneity due to TCP was evident. Two different AP powder sizes were used to make disks, 20 and 50 μm.

A reference measurement of near normal reflectance was made using a first-surface aluminum mirror. The normal reflectance for aluminum was calculated from the known spectral optical constants for aluminum[6] using

$$R_N = \frac{(n-1)^2 + k^2}{(n+1)^2 + k^2} \tag{1}$$

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